The Excel Math Competition

Sep 2025(Calculus Exam)

1. This is a 60 minute individual exam.
2. No collaboration or external devices (like a calculator) are are allowed.
3. The first 10 questions are worth [5] points each and are multiple choice.
4. The last 5 questions are worth [10] points each and are short response.
5. Each of the final 5 questions have answers which are positive integers between 000 and 999, inclusive.
6. The questions are arranged in roughly ascending difficulty.
7. All fractions are properly rationalized and simplified. For example, if your answer is in the form $\frac{a}{b}$ and you are asked to add $a + b$, it is assumed that a and b are coprime.
8. If you believe a question is seriously flawed, or have an answer which is not one of the listed answers, there will be a 10-minute dispute period after the test, after which no disputes will be accepted.
9. In the event of a dispute, **leave the question blank** and let your proctor know after the testing time ends.
10. All disputes will be considered on an individual-by-individual basis, so no student will receive credit if they did not submit a dispute, except for in the case of a question being thrown out.

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1 [5]

Its a known fact that James Doakes is the Bay Harbor Butcher. Doakes is taking care of business on the Miami coastline on his boat which is traveling at a velocity of $f(t)[\frac{m}{s}]$ eastward. What is his acceleration at time $t = 5$ minutes if $f(t) = t^2 - 4t + 57$

a) 596 $\frac{m}{s^2}$ west b) 600 $\frac{m}{s^2}$ east c) 604 $\frac{m}{s^2}$ east d) 608 $\frac{m}{s^2}$ east e) 596 $\frac{m}{s^2}$ east

2 [5]

$$L = \int_1^5 \frac{1}{x^2 - 5x + 6} dx$$

What is e^L

a) $\frac{1}{3}$ b) $\frac{1}{5}$ c) $\frac{1}{7}$ d) $\frac{1}{9}$ e) None of the Above

3 [5]

Lil Mike (a friend of the test writer) is obsessed with One Piece. Unfortunately, he is not a fan of the pacing of One Piece so he must watch One Pace (a version of One Piece with filler cut out of it). If $f(x)$ represents the amount of filler cut out of the series in hours at an instance of time and x is the number of years since the One Piece anime has released, then how much content has been cut in total for the entire show given it has been running for approximately 25 years.

$$f(x) = |x - 4|$$

a) $\frac{451}{2}$ Hours b) $\frac{453}{2}$ Hours c) $\frac{455}{2}$ Hours d) $\frac{457}{2}$ Hours e) $\frac{459}{3}$ Hours

4 [5]

$$\int_1^2 \frac{3x^2 - 2 + \frac{1}{2\sqrt{x}}}{x^3 - 2x + 7 + \sqrt{x}} dx = D$$

If D is of the form $\ln(\frac{A+\sqrt{B)}}{C})$, what is $A + B + C$?

a) 18 b) 20 c) 22 d) 24 e) 26

5 [5]

What can be said about the behavior of

$$\sum_{n=0}^{\infty} \frac{\sin(n)}{n}$$

a) Converges Absolutely b) Converges Conditionally c) Diverges d) Bifurcates e) None of the Above

6 [5]

Congratulations! You have opened your own pizzaria! You are currently making your own cheese pizza with the radius of only the cheese portion being r and the radius of the whole pizza being R (including the crust and the cheese). The cheesy part of the pizza costs \$0.05 per square inch and the crust costs \$0.02 per square inch. You sell to the customer the cheese pizza for \$0.10 per square inch of the cheesy part of the pizza and \$0.05 per square inch of the crusty part of the pizza. The radius R of your entire pizza is 8 inches and you can only spend a total of $\$(2.75\pi)$ creating the pizza, what is the most optimal thickness of the crust that maximizes profit for you?

a) 0 inches b) 1 inch c) 3 inches d) 5 inches e) 7 inches

7 [5]

$\lfloor x \rfloor$ is the greatest integer less then x

$\lceil x \rceil$ is the least integer greater then x

$$\int_1^\infty \frac{\lfloor x \rfloor}{x^2 \lceil x \rceil} dx$$

a) $\frac{\pi^2}{6} - \frac{1}{2}$ b) $\frac{\pi^2}{2} - 1$ c) $\frac{\pi^2}{3} - 1$ d) $\frac{\pi^2}{6} - 1$ e) $\frac{\pi^2}{2} - \frac{1}{2}$

8 [5]

$$y = b^x \cdot \frac{d}{dx}[b^x \cdot \frac{d}{dx}[b^x \cdot \frac{d}{dx}[...]]]$$

if $y(0) = 1$, find $\ln(y(\log_b \frac{3}{2}))$?

a) $\frac{1}{2\ln(b)}$ b) $\frac{1}{3\ln(b)}$ c) $\frac{1}{4\ln(b)}$ d) $\frac{1}{9\ln(b)}$ e) $\frac{1}{16\ln(b)}$

9 [5]

$$L = \lim_{n\to\infty} [\Pi_{k=0}^{n} (1 + \cos(\frac{k}{n})]^{\frac{1}{n}}$$

What is $\ln(L)$?

Use $\ln(1 + x) \approx x$

a) $\tan(1)$ b) $\cos(1)$ c) $\sin(1)$ d) $\sec(1)$ e) $\csc(1)$

10 [5]

$$L = \lim_{n\to\infty} (\frac{\binom{5n}{n}}{\binom{5n}{2n}})^{\frac{1}{n}}$$

a) 0 b) $\frac{27}{64}$ c) $\frac{64}{27}$ d) 1 e) $\frac{3}{2}$

11 [10]

$$\int (\sin(6x)\cos(7x)dx = -\frac{\cos(Ax)}{B} + \frac{\cos(Ex)}{F} + C$$

What is $2A + \frac{B}{2} + (EF)^4$

12 [10]

$$L = \int_1^3 -\frac{5x+6}{x^7+x^2+x}dx$$

What is e^L?

13 [10]

Find the area of the following region as $n \to \infty$:

$$(x^2+y^2)^n = 67xy^n$$

If A is the area, your answer should be $\frac{3\pi}{A}$

14 [10]

if $c = \frac{a}{b}$, what is $a+b$? k is an abritrary constant.

$$n!! = n \cdot (n-2) \cdot (n-4)...$$

$$\lim_{n\to\infty} \frac{c^n(3n)!}{(2n)!(4n-1)!!} = k$$

15 [10]

$$L = \int_0^{\frac{\pi}{2}} \sqrt{\tan x}dx$$

Find $\frac{12\pi^4}{L^4}$