The Excel Math Competition

September 2025 (Advanced Algebra/Geometry Exam)

1. This is a 60 minute individual exam.
2. No collaboration or external devices (like a calculator) are allowed.
3. The first 10 questions are worth [5] points each and are multiple choice.
4. The last 5 questions are worth [10] points each and are short response.
5. Each of the final 5 questions have answers which are positive integers between 000 and 999, inclusive.
6. The questions are arranged in roughly ascending difficulty.
7. All fractions are properly rationalized and simplified. For example, if your answer is in the form $\frac{a}{b}$ and you are asked to add $a+b$, it is assumed that a and b are coprime.
8. If you believe a question is seriously flawed, or have an answer which is not one of the listed answers, there will be a 10-minute dispute period after the test, after which no disputes will be accepted.
9. In the event of a dispute, **leave the question blank** and let your proctor know after the testing time ends.
10. All disputes will be considered on an individual-by-individual basis, so no student will receive credit if they did not submit a dispute, except for in the case of a question being thrown out.

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1 [5]

Welcome (or, welcome back!) to the September 2025 edition of the ExcelAcademe Math Competition! As you some of you may know, the Autumnal Equinox falls on September 22nd this year, which also happens to be the starting date of our testing window for this month! On this day, there are exactly 12 hours of night and 12 hours of day at the equator. In honor of this, Rayan collects 6 red leaves and 6 yellow leaves, and shuffles them randomly before lining them all up in a row. What is the probability that the number of red leaves in the first 6 positions is less than the number of red leaves in the last 6 positions?

a) $\dfrac{100}{231}$ b) $\dfrac{50}{231}$ c) $\dfrac{131}{231}$ d) $\dfrac{131}{462}$ e) $\dfrac{331}{462}$

2 [5]

Evaluate

$$\sum_{n=0}^{12} n^3$$

a) 78 b) 650 c) 5950 d) 6084 e) 60710

3 [5]

Compute the remainder when 5^{100} is divided by 7.

a) 0 b) 1 c) 2 d) 4 e) 6

4 [5]

Evaluate:

$$\sum_{n=0}^{\infty} n(\frac{1}{3})^n$$

a) $\frac{1}{2}$ b) $\frac{1}{3}$ c) $\frac{3}{4}$ d) $\frac{3}{8}$ e) $\frac{3}{16}$

5 [5]

David finds himself trapped in a magical room with 3 doors. One door leads to tunnel featuring a 10 minute walk to the outside. The second leads to a loop that takes 20 minutes to walk through before returning to the magical room. The third leads to a loop as well, which takes 30 minutes to walk through before returning to the magical room. Also, because the room is magical, each time David returns to the room, the doors are in a randomized order. What is the expected number of minutes it will take David to escape the room, given that he does not know which door leads outside but will continue to keep going through a random door whenever he finds himself back in the magical room?

a) 10 minutes   b) 20 minutes   c) 30 minutes   d) 45 minutes   e) 60 minutes

6 [5]

David has escaped the magical room, but is now in a mystical room! The mystical room has 3 doors. Once again, one door leads to a 10 minute walk to the outside, and another door leads to a 20 minute loop that returns to the mystical room. However, this time, the third door leads to a 6 minute walk to the secret room. The secret room has only 2 doors, one leading to the outside after an 18 minute walk, and the other leading back to the original mystical room after a 30 minute walk. If the doors are in a randomized order each time David returns to either room, what is the expected number of minutes it will take David to escape the mystical room?

a) 40 minutes   b) 54 minutes   c) 60 minutes   d) 72 minutes   e) 84 minutes

7 [5]

A bus travels west at a speed of 1 kilometer a minute on a perfectly straight road. However, there is also a circular storm that travels perfectly northwest (at a 45 degree angle west of north) at a speed of $\sqrt{2}$ kilometers per minute. At any given time t (in minutes), the storm has a radius of $\frac{t}{2}$. At time $t = 0$, the storm is 25 kilometers south of the bus. At time $t = t_1$ minutes, the bus enters the storm circle. At time $t = t_2$, the bus leaves the storm circle. Find $t_1 + t_2$ in minutes.

a) $\frac{50}{3}$ b) $\frac{100}{3}$ c) $\frac{150}{3}$ d) $\frac{200}{3}$ e) $\frac{250}{3}$

8 [5]

Define the range of function $P(n)$ as the set of integers k of base n such that $n^3 \leqslant k \leqslant n^4$. Let x represent the set of integers within the range of $P(9)$. Find the probability that for all integers k in x, $k \bmod 18 \equiv 1$.

a) $\frac{1}{30}$ b) $\frac{432}{8887}$ c) $\frac{324}{7889}$ d) $\frac{576}{9999}$ e) $\frac{1}{24}$

9 [5]

An equilateral triangle with side length $n > 2$ is split into n separate triangles of equal area within the original one. All but two of the new triangles are highlighted gray. This process continues with each of the remaining two uncolored triangles, such that for each of them, all but two of the new triangles are highlighted gray. This goes on infinitely. What is the combined area of all the gray regions, in terms of n?

a) $\frac{n^2+8n+16}{n^2+4n+4}$ b) $\frac{\sqrt{3}(n^3-4n^2)}{4n-8}$ c) $\frac{n^2\sqrt{3}}{4}$ d) $\frac{n^3\sqrt{2}-2n}{8n-4}$ e) n^2

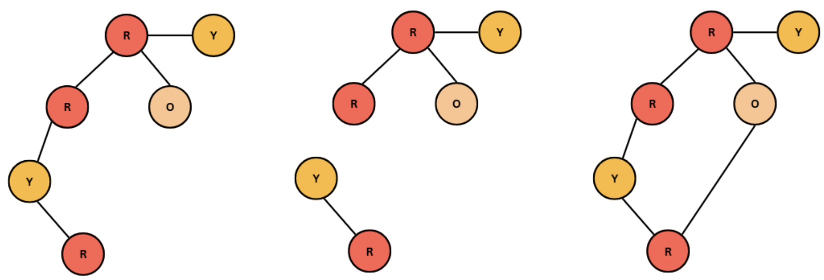
10 [5]

$\triangle ZAP$ is a right triangle with right angle $\angle A$. 3 Cevians, with lengths p,q, and r, are drawn from $\angle A$ to the hypotenuse ZP so that the hypotenuse is split into 4 line segments, each having a length of 2. Find $p^2 + q^2 + r^2$

a) 20 b) 36 c) 48 d) 56 e) 64

11 [10]

Rayan is back at it with collecting leaves. He collects 2 red leaves, 1 yellow leaf, and 1 orange leaf, and decides to treat each leaf like a "node" and makes connections between them with branches, creating "trees". A tree is, in this context, an arrangement of leaves connected with branches such that between any two pairs of distinct leaves, there is exactly one way to go from one leaf to another. How many distinct trees can he make, if every leaf of the same color is indistinguishable?



Tree **NOT a Tree** **NOT a Tree**

12 [10]

To celebrate the start of the September EMC lining up with the Autumnal Equinox, help me with the following problem. Call a positive integer equinox-balanced if the sum of its even digits equals the sum of its odd digits. How many equinox-balanced integers are there between 1000 and 1999?

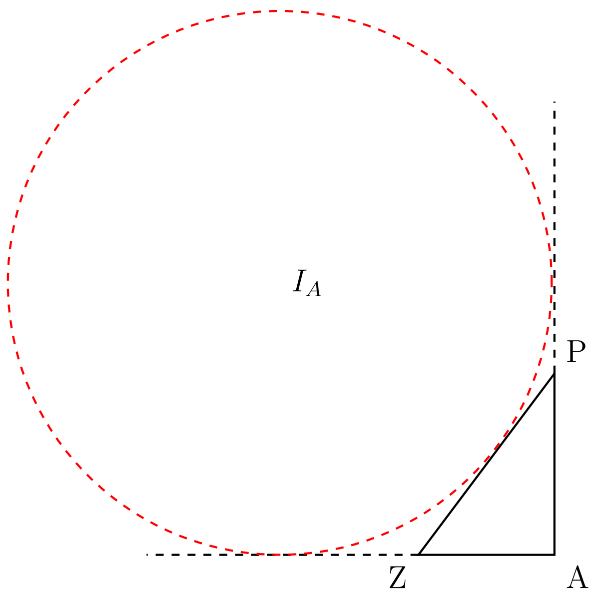
13 [10]

Let $RYAN$ be an isosceles trapezoid with bases $RY = 21$ and $AN = 100$. Suppose $RN = YA = x$, and there is a circle with center on AN that is tangent to both RN and YA. Determine the last 3 digits of the square of the smallest possible value of x.

14 [10]

You may have learned about the four main centers of a triangle: the incenter, circumcenter, orthocenter, and centroid. These are not the only centers a triangle has, as there are thousands of other ones as well. The one we will be concentrating on in this problem is a triangle's three ex-centers, which is found by extending each side of a triangle and creating circles that are tangent to two of the extended sides and the third side.

Let $\triangle ZAP$ be a right triangle where $ZA = 3$, $AP = 4$, and $ZP = 5$. What is the ex-radius of the triangle opposite to vertex A? (Hint: To derive the formula for ex-radius, just find the area of the triangle and a part of the excircle and then subtract the extra area you have)



15 [10]

Suppose x_1, x_2, x_3, x_4, x_5 are non-negative real numbers such that $x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 = 3$. The minimum possible value for

$$2x_1^3 + 3x_2^3 + 6x_3^3 + 9x_4^3 + 54x_5^3$$

is $\frac{A\sqrt{B}}{C}$, what is the remainder when $(A + B - C)$ is divided by 1000