The Excel Math Competition

Sep 2025(Introductory Algebra/Geometry Exam)

1. This is a 60 minute individual exam.
2. No collaboration or external devices (like a calculator) are are allowed.
3. The first 10 questions are worth [5] points each and are multiple choice.
4. The last 5 questions are worth [10] points each and are short response.
5. Each of the final 5 questions have answers which are positive integers between 000 and 999, inclusive.
6. The questions are arranged in roughly ascending difficulty.
7. If you believe a question is seriously flawed, or have an answer which is not one of the listed answers, there will be a 10-minute dispute period after the test, after which no disputes will be accepted.
8. In the event of a dispute, **leave the question blank** and let your proctor know after the testing time ends.
9. All disputes will be considered on an individual-by-individual basis, so no student will receive credit if they did not submit a dispute, except for in the case of a question being thrown out.

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1 [5]

Over the summer, Omar's parents decided to move to California from Florida, and he unfortunately had to switch schools and make his own schedule! Omar needs to take five core classes (science, english, math, history, and art), as well as an elective and PE. Given that Omar wants to take art, PE, and history all in a row, how many different schedules can Omar create, assuming no constraints on his schedule otherwise?

a) 36 b) 72 c) 120 d) 144 e) 180

2 [5]

Omar's issues didn't end there, however. He decided to take the hardest classes offered by his new school and has stayed up every night for a month. Last night, he looked at the clock while studying and saw it was 3:06! Of all the times from 12:00 to 11:59, how many unique times are there so that hours divide the minutes? (To clarify, if the time is expressed as $hh : mm$, how many ways are there so that hh is a factor of mm?)

a) 1672 b) 180 c) 188 d) 192 e) 196

3 [5]

$$f(x) = x^2 + 3x + 1$$
$$g(x) = x^2 + x + 5$$

What is $f(g(1)) - g(f(1))$ equal to?

a) -106 b) -36 c) 0 d) 36 e) 106

4 [5]

Suppose that $A * B$ is defined as $A^2 - 3AB + 2B^2$. Find $3 * (1 * 2)$

a) 0 b) 9 c) 12 d) 18 e) 28

5 [5]

If the area of a square can be expressed as the polynomial:

$$A(x) = 9x^2 + 24x + 16$$

where x is the side length of the square, what is the perimeter of the square in terms of x?

a) $3x^2$ b) $3x + 4$ c) $6x + 8$ d) $12x + 16$ e) $3x^2 + 4$

6 [5]

$$x + \frac{1}{x} = 4$$

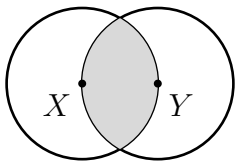
Give the value of L if:

$$(x^3 + \frac{1}{x^3}) - (x^2 + \frac{1}{x^2}) = L$$

a) 38 b) 42 c) 46 d) 50 e) 54

7 [5]

Suppose circles X and Y both have equal radii $R = 4$. The center of circle X is on the circumference of Y and the center of circle Y is on the circumference of Y. What is the area of their area of intersection (shaded region)?



a) $\frac{16\pi}{3} - 4\sqrt{3}$ b) $\frac{32\pi}{3} - 4\sqrt{3}$ c) $\frac{16\pi}{3} - 8\sqrt{3}$ d) $\frac{32\pi}{3} - 8\sqrt{3}$ e) $\frac{32\pi}{3}$

8 [5]

How many asymptotes does the following graph have:

$$f(x) = \frac{x^2 - 5x + 6}{x^2 - 6x + 8}$$

a) 1 b) 2 c) 3 d) 4 e) 5

9 [5]

Let r_1, r_2, r_3, r_4, r_5 represent the roots of the following polynomial:

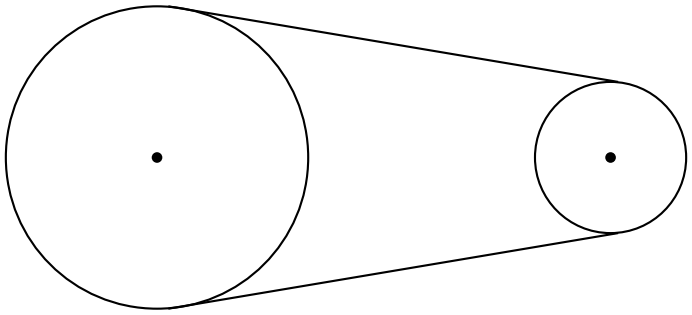
$$f(x) = x^5 - 4x^4 + 8x^3 + 3x^2 + 2x + 11$$

What is $\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \frac{1}{r_4} + \frac{1}{r_5}$?

a) $-\frac{1}{11}$ b) $\frac{4}{11}$ c) $-\frac{8}{11}$ d) $\frac{3}{11}$ e) $-\frac{2}{11}$

10 [5]

Two pulleys are attached to each other via belt (picture not drawn to scale). The radius of the smaller circle is $r = 3$ and the radius of the larger circle is $R = 13$ and the center to center distance between these two pulleys is 20 units. What is the total length of the belt then wraps around these two pulleys?



a) $\frac{58\pi}{3}$ b) $\frac{56\pi}{3} + 10\sqrt{3}$ c) $\frac{58\pi}{3} + 10\sqrt{3}$ d) $\frac{56\pi}{3} + 20\sqrt{3}$ e) $\frac{58\pi}{3} + 20\sqrt{3}$

11 [10]

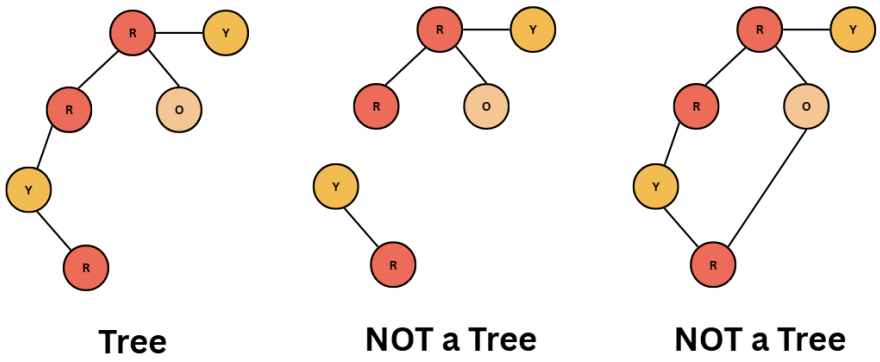
What is the sum of all integers less then or equal to 100 that have an odd number of factors?

12 [10]

David finds himself trapped in a magical room with 3 doors. One door leads to tunnel featuring a 10 minute walk to the outside. The second leads to a loop that takes 20 minutes to walk through before returning to the magical room. The third leads to a loop as well, which takes 30 minutes to walk through before returning to the magical room. Also, because the room is magical, each time David returns to the room, the doors are in a randomized order. What is the expected number of minutes it will take David to escape the room, given that he does not know which door leads outside but will continue to keep going through a random door whenever he finds himself back in the magical room?

13 [10]

Rayan is collecting leaves. He collects 2 red leaves, 1 yellow leaf, and 1 orange leaf, and decides to treat each leaf like a "node" and makes connections between them with branches, creating "trees". A tree is, in this context, an arrangement of leaves connected with branches such that between any two pairs of distinct leaves, there is exactly one way to go from one leaf to another. How many distinct trees can he make, if every leaf of the same color is indistinguishable?



14 [10]

To celebrate the start of the September EMC lining up with the Autumnal Equinox, help me with the following problem. Call a positive integer equinox-balanced if the sum of its even digits equals the sum of its odd digits. How many equinox-balanced integers are there between 1000 and 1999?

15 [10]

You may have learned about the four main centers of a triangle: the incenter, circumcenter, orthocenter, and centroid. These are not the only centers a triangle has, as there are thousands of other ones as well. The one we will be concentrating on in this problem is a triangle's three ex-centers, which is found by extending each side of a triangle and creating circles that are tangent to two of the extended sides and the third side.

Let $\triangle ZAP$ be a right triangle where $ZA = 3$, $AP = 4$, and $ZP = 5$. What is the ex-radius of the triangle opposite to vertex A? (Hint: To derive the formula for ex-radius, just find the area of the triangle and a part of the excircle and then subtract the extra area you have)