The Excel Math Competition

May 2025 (Calculus Exam)

1. This is a 60 minute individual exam.
2. No collaboration or external devices (like a calculator) are allowed.
3. The first 10 questions are worth [5] points each and are multiple choice.
4. The last 5 questions are worth [10] points each and are short response.
5. Each of the final 5 questions have answers which are positive integers between 000 and 999, inclusive.
6. The questions are arranged in roughly ascending difficulty.
7. If you believe a question is seriously flawed, or have an answer which is not one of the listed answers, there will be a 10-minute dispute period after the test, after which no disputes will be accepted.
8. In the event of a dispute, **leave the question blank** and let your proctor know after the testing time ends.
9. All disputes will be considered on an individual-by-individual basis, so no student will receive credit if they did not submit a dispute, except for in the case of a question being thrown out.

Test writer:
David Zapata
Joseph Levenston

Test Editor:
David Zapata

Problem Contributors:
David Zapata
Joseph Levenston
Siyuan Du

Sponsors:

1 [5]

Welcome to the Excel Academe Math Competition! This is our first time doing a calculus test ever, so congratulations on being a part of this historical moment! We hope everyone did good on their AP exams! The following equates to a simplified fraction of the form $\frac{m}{k}$. What is $m + k$?

$$\left.\frac{d(5x^2 + 18x + 2008)}{d(e^{2025x})}\right|_{x=0}$$

a) 2024 b) 2025 c) 2030 d) 2043 e) 4033

2 [5]

Find the slope of the normal line of $x^3 + 2x + 5$ at $x = 1$

a) $-\frac{1}{5}$ b) $\frac{1}{5}$ c) -5 d) 5 e) 0

3 [5]

David is running on 30 minutes of sleep again and has Calculus homework due at 11:59PM (it is currently 11:50PM), solve this integral so David can get a 100% on his homework.

$$\int_0^1 \ln(x)$$

a) e b) -1 c) 0 d) 1 e) e

4 [5]

If $f(x) = e^{3x-3} + 5x^2 - 6$, find the derivative of $f^{-1}(x)$ at $x = 0$

a) $-\frac{1}{13}$ b) $\frac{1}{13}$ c) 13 d) -13 e) 0

5 [5]

$$\int \frac{\sqrt{x^2 + 16}}{x^4} = -\frac{(x^\alpha + \beta)^\gamma}{\lambda \cdot x^\rho} + C$$

What is $\alpha + \beta + 2 \cdot \gamma + \lambda + \rho$?

a) 72 b) 81 c) 90 d) 99 e) 108

6 [5]

$$\lim_{n\to\infty} n^{\frac{2009}{2}} \cdot \int_0^{\frac{1}{\sqrt{n}}} x^{5x^2+18x+2008} dx = ?$$

a) 0 b) $\frac{1}{2007}$ c) $\frac{1}{2008}$ d) $\frac{1}{2009}$ e) DNE

7 [5]

An important concept to understand in the study of classical physics is the center of mass of an object with an non-uniform density. For example, if we were given a function for r in terms of mass in the x direction, we can find the center of mass in the x direction by using the formula $x_{cm} = \frac{\int r \cdot dm}{\int dm}$, Suppose we had a strictly 1 dimensional linear rod that is 5 meters long and has a function that gives the density at a specified length which is $\lambda(x) = x^2 + 3\sqrt{x}$ where $x = 0$ is the to the left of the rod. Let x_{cm} be the center of mass of this rod and let M be the total mass of this rod. $|x_{cm}| - M = b\sqrt{5} + \frac{m}{n}$. What is $b + m + n$?

a) 1407 b) 1537 c) 1497 d) 1337 e) 1607

8 [5]

In statistics, a probability density function (PDF) allows us to calculate the probability of a continuous random variable falling within a specific interval by finding the area under the curve of the PDF between the chosen bounds and the x-axis. The probability density function of the standard normal distribution (a normal distribution with a mean of 0 and a standard deviation of 1) is:

$$\frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{x^2}{2}}$$

Using this knowledge, answer the following:

$$\int_{-\infty}^{\infty} e^{-x^2} dx$$

a) $\sqrt{\frac{\pi}{2}}$ b) $\sqrt{\frac{3\pi}{2}}$ c) $\sqrt{2\pi}$ d) $\sqrt{e}$ e) $\sqrt{\pi}$

9 [5]

$$\int_0^1 e^{-x^2} dx + \int_{-\sqrt{e}}^{\sqrt{e}} \frac{\sin(x)}{1 + x^2 + \cos(x)} dx + \int_1^{e^{-1}} \sqrt{\ln(\frac{1}{x})} dx = e^k$$

What is k?

a) -2 b) -1 c) 0 d) 1 e) 2

10 [5]

$$\int_0^2 x \cdot \sqrt{x \cdot \sqrt[3]{x \cdot \sqrt[4]{x \dots}}}\, dx$$

a) $\frac{2^{e+2}}{e+2}$ b) $\frac{2^{e+2}}{e}$ c) $\frac{2^e}{e}$ d) $\frac{2^e+1}{e}$ e) $\frac{2^e+1}{e-2}$

11 [10]

If the area of the inner loop of the polar curve $r = 1 + 2\cos\theta$ is equal to $\pi - \frac{A\sqrt{B}}{C}$ where the fraction is in simplest form and properly rationalized, what is $A \cdot B \cdot C$?

12 [10]

$$\text{If } I_n = \int_{\frac{n}{2}}^{\infty} \binom{n}{x}\, dx, \text{ then find } \sum_{n=0}^{\infty} \frac{1}{I_n}$$

13 [10]

$$\int_1^2 \frac{3x^4 + 4x + 6x^{-1}}{x^6 + 3x^5 + x^2 + 1} dx = \ln(\frac{A}{B})$$

What is $A + B$ given that $\frac{A}{B}$ is a fraction in simplest form.

14 [10]

$$S = \sum_{n=1}^{\infty} \frac{\cos(n)}{n}$$

What is $\lfloor 100S \rfloor$?

15 [10]

$$L = \int_{-1}^{1} \sqrt{\frac{x-1}{x+1}} \cdot \arccos(\mathrm{x})$$

If $L = \frac{\pi^a}{b} + c$, what is a^{b^c}