The Excel Math Competition

Aug 2025(Advanced Algebra/Geometry Exam)

1. This is a 60 minute individual exam.
2. No collaboration or external devices (like a calculator) are are allowed.
3. The first 10 questions are worth [5] points each and are multiple choice.
4. The last 5 questions are worth [10] points each and are short response.
5. Each of the final 5 questions have answers which are positive integers between 000 and 999, inclusive.
6. The questions are arranged in roughly ascending difficulty.
7. If you believe a question is seriously flawed, or have an answer which is not one of the listed answers, there will be a 10-minute dispute period after the test, after which no disputes will be accepted.
8. In the event of a dispute, **leave the question blank** and let your proctor know after the testing time ends.
9. All disputes will be considered on an individual-by-individual basis, so no student will receive credit if they did not submit a dispute, except for in the case of a question being thrown out.

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1 [5]

Welcome everyone to the August edition of the ExcelAcademe Advanced Math Competition! It's been a while since I wrote one of these, so I may have forgotten some basic arithmetic :(

$$2 + 2 \times 2 + 2 \times 2 \times 2 = 12$$

You want to make the above equation true by replacing either +'s with ×'s or ×'s with +'s. Let n be the number of replacements (not counting flip-flops like $+ \to \times \to +$) necessary to accomplish this. What is the sum of all possible n? Answer 0 if no n exists.

a) 0 b) 3 c) 10 d) 15 e) 21

2 [5]

Okay I'm getting back into the groove here, but I'm still struggling with geometry... and I've unfortunately forgotten the value of π while sketching the Jane Street logo.

Fortunately, I only need to be within 1% of the true value of π in order to secure our Jane Street sponsorship money. I know π starts with a "3.", so I will guess every digit past the decimal uniformly at random and hope I get close enough! If p is the probability that I secure the sponsorship money, what is $\lfloor 100p \rfloor$?

a) 0 b) 2 c) 4 d) 6 e) 8

3 [5]

I've recently learned that number theory is quite a difficult topic. If you wish to taste Jim's Fowler, you must answer the following riddle!

Joshua writes the integers from 2 to 50, inclusive, on the chalkboard in Towers 112. He then chooses a positive integer n and writes the remainder when n is divided by each number on the chalkboard. Joshua notices that each of these remainders are nonzero and distinct from the rest!

Find the sum of the smallest and largest prime factors of the smallest possible value of $n + 1$.

a) 40 b) 43 c) 49 d) 53 e) 54

4 [5]

Find the minimum value of the following over reals x, y:

$$\sqrt{(x+2)^2 + (y+0)^2} + \sqrt{(x-2)^2 + (y-5)^2}$$

a) 0 b) $\sqrt{39}$ c) $\sqrt{41}$ d) $\sqrt{43}$ e) $\sqrt{57}$

5 [5]

I have 3 shapes of variable size: a circle, a square, and an equilateral triangle. I assign each to either P_1, P_2, or P_3. I then inscribe P_1 in P_2 in some orientation, and then circumscribe P_3 about P_2 in some orientation.

Out of all possible assignments of shapes to P_i and all possible orientations, what is the **minimum** value of $\left(\frac{R_3}{R_1}\right)^2$, where R_i is the circumradius of P_i?

a) $\frac{12+3\sqrt{3}}{16}$ b) $\frac{6+3\sqrt{3}}{8}$ c) $\frac{14+8\sqrt{3}}{9}$ d) $\frac{6+3\sqrt{3}}{2}$ e) 4

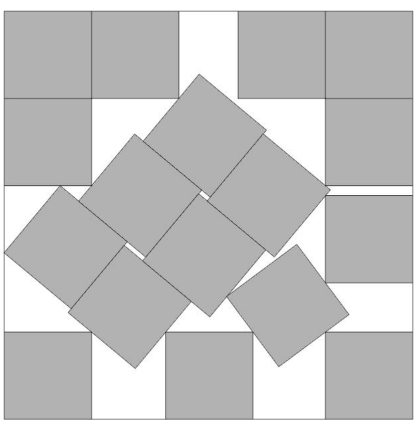
6 [5]

Gar-Bear slipped and dropped the $xy-$plane, causing every point (x, y) to travel to the point $(2x + 3y, x + 4y)$!

If before he slipped, Gar-Bear drew the graph of $x^2 + 4x + y^2 - 10y + 4 = 0$, what is the area enclosed by the graph after the fall?

a) 5π b) 10π c) 25π d) 50π e) 125π

7 [5]



The above picture is the current best known solution to the $n = 17$ case of a problem known as square packing, where the objective is to find the smallest side length of a square into which n unit squares can be fit without overlapping.

Find the minimum side length of a square into which 10 unit squares can be fit without overlapping (Hint: try the $n = 5$ case first, then just add some extra squares!)

a) $1/2 + \sqrt{5}$ b) $2\sqrt{2}$ c) $3 + \sqrt{2}/2$ d) $\sqrt{10}$ e) 4

8 [5]

A fourth degree polynomial $P(x)$ with integer coefficients has roots at $-1, 3$, and 5. David couldn't find the fourth root, so he picked a random integer instead. Which of the following is **not** a possible value of the sum of the coefficients of $P(x)$?

a) 0　　b) $2^4 - 2^3$　　c) $2^8 - 2^7$　　d) $2^{64} - 2^{63}$　　e) $2^{128} - 2^{127}$

9 [5]

Graham has 13 crackers numbered 1 through 13. Tintu will choose Graham's crackers at random one at a time until the sum of the numbers on her crackers is larger than the sum of the numbers on Graham's remaining crackers. How many crackers, on average, will Graham have left once this occurs?

a) 5　　b) 5.5　　c) 6　　d) 6.5　　e) 7

10 [5]

Zappy picks two primes p, q. He notices that there are 12 distinct complex numbers z satisfying both $\dfrac{z^{pq}-1}{z^p-1} = 0$ and $\dfrac{z^{pq}-1}{z^q-1} = 0$. Find the sum of the possible distinct values of pq.

a) 21　　b) 47　　c) 57　　d) 67　　e) 71

11 [10]

Siyuan secretly swoons over sequences and series. Today he has discovered a sequence of positive real numbers $\{\theta_n\}_{n=1}^{\infty}$ subject to the following:

$$\theta_{n+2} = \frac{\theta_{n+1} + \theta_n}{1 - \theta_{n+1}\theta_n}; \qquad \theta_1 = \theta_2 = \tan(1)$$

This sequence was too triggy for Siyuan to understand, so he made the set S of the unique integers k_i for which $\tan(k_i) = \theta_i$, with $1 \leqslant i \leqslant 2025$. Help Siyuan by finding the number of elements of S which are divisible by 3.

12 [10]

Jiwu appeared in my dream last night and presented me with the following expression, but I seem to have forgotten what it evaluates to... He hinted to me that it requires the use of the identity $(n+1)^2 = 1 + n(n+2)$, but I'm not sure how that helps!

Find the value of

$$\sqrt{1 + 67\sqrt{1 + 68\sqrt{1 + 69\sqrt{\dots}}}}$$

13 [10]

There exist 2025 positive real numbers whose sum is 2026 and whose reciprocals sum to 2026. If x is one of these real numbers, then the maximum possible value of $x + \dfrac{1}{x}$ is the simplified fraction $\dfrac{a}{b}$. Find the remainder when $a + b$ is divided by 1000.

14 [10]

Call an integer-coefficient polynomial P $m-shy$ if for any integer n, $P(n) \equiv 0 \pmod{m}$, and at least one coefficient of P is not 0 mod m.

Find the number of $6-$shy polynomials with coefficients in $[0, 6)$ and degree less than 5.

15 [10]

Triangle $\triangle ABC$ has incenter I and circumcenter O. You know that $\overline{AI} = \overline{AO}$. What is the **maximum** possible value of $\angle A$ in degrees?